JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 14, No. 4, October–December 2000

Estimation Methods for Two-Dimensional Conduction Effects of Shock–Shock Heat Fluxes

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Highly localized heating rates occur in extreme thermal environments, such as shock–shock interaction at hypersonic speeds. Experimental estimation of heat flux usually involves measurement of discrete unsteady surface temperatures on low conductivity materials that are chosen to reduce conduction effects. Typically, temperature measurements are reduced to heat fluxes using one-dimensional conduction techniques. However, lateral conduction from localized high heat flux regions into low heat flux regions is significant and influences the one-dimensional solution. The one-dimensional solution also suffers mathematical instabilities. To evaluate the nonuniform, unsteady surface flux from measured temperatures, an inverse technique was devised that damps instabilities in the temporal direction and resolves large and sudden flux changes in the spatial direction. Based on previous work, a simple inverse method was used in time, and a function specification method was used in space. Furthermore, a technique was devised to expedite the solution by marching in space as well as in time. The new multidimensional inverse method was found to resolve steep spatial gradients more accurately in flux than a one-dimensional method. Furthermore, the inverse procedure exhibits better stability than a multidimensional forward technique.

Nomenclature

C_h	=	normalized heat flux; Stanton number
$\boldsymbol{H}$	=	order of regularization matrix, dimensionless
q	=	heat flux, W/cm^2
R	=	regularization term
S	=	objective function, K^2
T	=	calculated temperature, K
t	=	time, s
$\boldsymbol{X}$	=	sensitivity matrix
Y	=	measured temperature, K
α	=	regularization parameter, K^{2}cm^4/W^2
Ψ^{-1}	=	matrix of measurement variances, 1/K^2

Subscripts

i, j	=	temporal index
n	=	normalized
s	=	space
t	=	time
0	=	current iteration

Introduction

SHOCK–SHOCK interactions during hypersonic flight can produce very intense and very damaging localized heat flux. Damaging thermal environments characterized by high temperatures and localized heating rates can be created during high-speed flight and must be studied to prevent mission failure. An example of such damage was observed on the X-15.[1,2] Another example from the previous National Aerospace Program (NASP) is that shock–shock heat flux on the engine's cowl leading edge was estimated with computational fluid dynamics (CFD) to be as much as 5.6×10^{-8} W/m^2 (50,000 Btu/ft^2 s) (Ref. 3).

Shock–shock interaction studies have been performed for many years beginning with leading-edge interference experiments[4] and the characterization of shock interactions by Edney.[5] The significance of the potential damage and necessity for designs that account for these thermal phenomena has been the subject of numerous studies, such as the work by Weiting and Holden.[6] Experimental and CFD characterization of shock–shock interaction heating was done at NASA Langley Research Center and through funding by NASA Langley Research Center under the NASP effort in an extensive experimental program at Calspan University at Buffalo Research Center. In one set of experiments from the latter effort,[7] shock–shock interactions produced a 30-times amplification of heat flux on a 3-in.-diam cylinder; 25% of the intense peak occurred in a region of only 4 circumferential degrees. The measurements were made on a Pyrex TM 7740 substrate and included only one-dimensional conduction. Recently, more complex shock interactions have been studied such as those examined by Berry and Nowak[8] and Neumann.[9]

Conclusions shared by all of these experiments are that the heating generated by shock interaction has very high gradients and that the heating rates are extremely large and localized. Furthermore, oscillating levels of heat fluxes can be found at different positions on models subjected to shock interaction heating. The reader is referred to work by Edney[5] and Glass et al.[10] for an analysis of the flow patterns produced in shock interactions that are responsible for the heating rates characteristic of shock–shock interactions. CFD work by several researchers, for example, Prabhu[3] and Wright et al.,[11] has since verified the character of the shock–shock heating flux amplifications found experimentally.

To evaluate the nonuniform heating rates, data reduction schemes have been used to convert measured temperatures from the surface of models to heat fluxes. This problem is unstable,[12,13] and many methods have been proposed as solution methodologies. Walker and Scott[14] provide a summary of commonly used methods and their strengths and weaknesses. All currently used methods assume

Received 1 November 1999; revision received 29 May 2000; accepted for publication 1 June 2000.

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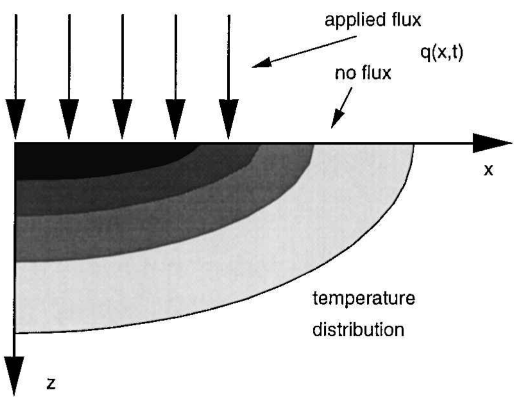

Fig. 1 Temperature distribution due to a step flux (after some time) indicates that the temperature in the nonheated region will change.

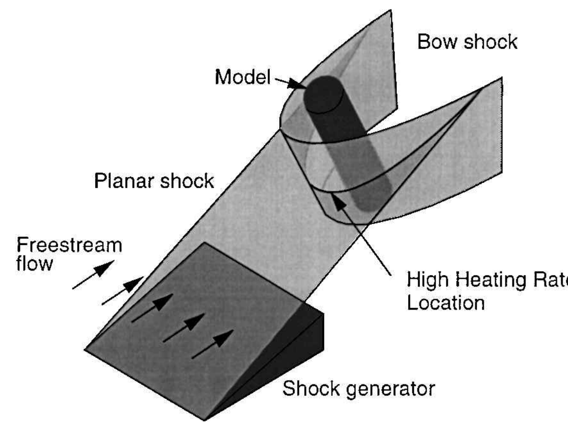

Fig. 2 Experimental setup shows where the planar shock and bow shock will intersect and produce the high heating rates.

a one-dimensional conduction model. However, a simple analysis can demonstrate the fallacy in this assumption.

Suppose a heat flux is being applied to a surface in a step fashion such that there is some area that sees a zero flux. We know that the heat being added will diffuse into the nonheated region, causing the temperature to rise there as exemplified in Fig. 1. A one-dimensional data reduction routine will result in a nonzero flux estimate for any sensor that sees a temperature change. Therefore, we must use a two-dimensional conduction model to describe accurately a spatially dependent heating rate where the heat is diffusing laterally. The difference between one-dimensional and two-dimensional analysis depends on the specific problem and is what will be identified in this work.

A direct solution to the data reduction would be to calculate the multidimensional conduction solution using the measured temperature as a boundary condition. The flux could then be found by differentiating the resulting temperature distribution (normal to the surface) at the surface and using Fourier's law. Direct methods, in general, have several inherent disadvantages, as described by Walker and Scott.[15] For example, they tend to display unstable behavior resulting from differentiation of discrete data. Furthermore, we have to make some assumption about the temperature distribution between sensors and between time steps for these approaches. In effect, we are including high-frequency information that cannot possibly appear in discrete data, which reduces our confidence in the solution methodology.[16]

For an inverse approach we must first decide how the heat flux (not temperature) is to vary between sensors and between time steps. For most cases a linear change in heat flux between estimation points is assumed. The term estimation points is used here to mean the position and time where and/or when a flux is estimated. Normally this will coincide with the sensors and the measurements; however, we have suggested the potential to estimate fluxes with a higher spatial resolution than the sensor spacing used to record actual measurements through an inverse technique. This approach was first examined by Walker and Scott,[14] where two-dimensional inverse techniques were compared to two-dimensional forward techniques.

The goal of this work is to examine the problem of lateral conduction effects in estimating heat fluxes from surface temperature measurements and to introduce a solution technique that can resolve fluxes that are spatially dependent. This work is a direct application of the methodologies discussed in a previous work by Walker and Scott[17] and described earlier. However, the methods used have only previously been applied to one-dimensional problems. This work represents a first attempt to solve nonuniform high heat flux problems with the new method. A brief description of inverse solution techniques will be given, as well as some results from an implementation of the inverse technology with experimental test data. These results will also be compared to a one-dimensional forward technique that is considered state of the art.[18]

Experiment Description

The shock interaction temperature data used for this study were obtained from NASA Langley Research Center; the test was documented by Berry and Nowak.[8] A three-dimensional flow pattern was generated with a planar incident shock impinging a cylinder that was arranged in a plane perpendicular to the shock (Fig. 2). The interaction between the planar shock and the bow shock produced supersonic jets ($M \approx 6$) that resulted in heating on the surface of the model at the point of impingement. The temperature history was measured using thin-film temperature gauges arranged along the length of the cylinder.

Two model configurations were used to examine the lateral conduction effects and localized jet resolution. In the first case, the temperature sensors were deposited directly onto the 1.27-cm-diam ceramic model. The sensors were placed as close as possible (limited by the application method) at a distance of 0.635 mm. To increase the sensor spacing on the model, a second application method was used. In this case, the sensors were sputtered onto a flat thin sheet of Upilex™ (a polyimidelike material, Ube Industries, Ltd). With this method, the spacing between gauges was reduced to 0.381 mm, which was limited primarily by the spacing requirements of the electrical leads to the sensors. The Upilex film was then bonded to the cylindrical model with high-temperature epoxy.

Approximately 90 channels of temperature measurements were collected at a sample rate of 50 Hz for each sensor for a duration of approximately 5 s. Even though the flow conditions were close to steady, small fluctuations such as vibrations due to injection of the model and impact of the flow result in small perturbations. Additionally, shock–shock interaction studies have been shown to be inherently unsteady. However, this particular experiment was shown to produce relatively stable measurements.[8] Therefore, the experiment with laminar flow conditions was expected to produce a relatively constant heat flux coefficient in time, and so a high sampling rate was not necessary.

Solution Methodology

The inverse heat conduction solution, which is being proposed as a solution to the data reduction problem, attempts to identify a boundary flux or heat transfer coefficient given an interior temperature measurement. Because measurements are discrete and contain noise, the solution is unstable, meaning that small uncertainties in the measurement result in large changes in the boundary estimate. In the present work, surface measurements are recorded that reduce the uncertainty in the solution somewhat compared to interior measurements.

The inverse solution estimates the boundary condition that produces a calculated response matching the measured response by employing an optimization routine. In other words, the difference between measured and calculated values is minimized. This can be performed by any number of optimization routines; in this case a

Newton's method was used. Inverse methodologies help stabilize the solution because 1) we do not have to take derivatives of the experimental data and 2) the precise matching of experimental data can be relaxed. The first point can very easily be accomplished by finding a boundary solution that minimizes the calculated and measured temperatures as in a least-squares fit. The second point is realized by requiring the residual (difference between calculated and measured values) to lie within the level of measurement noise, but not to match exactly. This suggests that some bias will be introduced into the solution through the methodology. There are two primary methodologies that will be used to perform the biased minimization.

The techniques presented here have been rigorously tested against known data and fabricated heat fluxes designed to mimic shock interaction data.[17] Furthermore, the methods have been verified with experiments that collect heat flux and temperature data simultaneously.[19]

Function Specification

The first method examined is one that has been applied to this problem previously using several test cases with simulated data.[17] It predicts a value for the heat flux, provided the estimate fits a predetermined functional form. We are, in essence, estimating the coefficients of a function that describe the boundary flux. Note that the estimates are found in a sequential manner such that only the current time and a few future times are examined simultaneously. Once the current estimate is determined, the procedure progresses to the next time step with its future measurements. By choosing the appropriate number of future time steps, we can use a first-order boundary approximation without introducing overwhelming bias. The appropriate number of future times for our case is one, the minimum required to fully define a line.

We have described the typical one-dimensional inverse solution technique. However, the goal is to enhance the solution methodology to include multidimensional conduction. Therefore, we must describe the implementation of the function-specification method in the spatial direction. Again, a functional form of the boundary flux (which is now a function of location as well as time) must be predetermined, and the coefficients that minimize the objective function must be found. Because the spatial direction is elliptical rather than parabolic in nature, the inclusion of neighboring sensors in the estimation procedure will help introduce a bias into the solution much like the future time steps did for the temporal direction. However, the solution cannot march in the spatial direction as it did in the temporal direction. As a result, all spatial estimates for a given time must be found simultaneously.

The formulation for the function coefficients that make up the estimates is found from the objective function, which is the sum of squares of the difference between measured, $\boldsymbol{Y}$, and calculated, $\boldsymbol{T}$, temperatures given by

$$S = (\boldsymbol{T} - \boldsymbol{Y})^T(\boldsymbol{T} - \boldsymbol{Y}) \tag{1}$$

that is to be minimized. Here, the components of the temperature vectors consist of temperatures from different sensors at a single time followed by the temperature for the same sensors at the next time step. For example, if we use subscripts to denote spatial location and superscripts to denote time step, considering two time steps and two spatial locations, the temperature vectors $\boldsymbol{T}$ and $\boldsymbol{Y}$ can be written as

$$\boldsymbol{T} = \left\{T_1^1,\ T_2^1,\ T_1^2,\ T_2^2\right\}^T \tag{2}$$

Note that the calculated temperatures are found from any legitimate conduction solution.

To minimize this objective function S, the derivative with respect to the flux can be set equal to zero. The boundary condition is introduced into the formulation by writing the first-order Fourier series expansion of the temperature as

$$\boldsymbol{T} = \boldsymbol{T}_0 + \boldsymbol{X}\boldsymbol{q}_0 \tag{3}$$

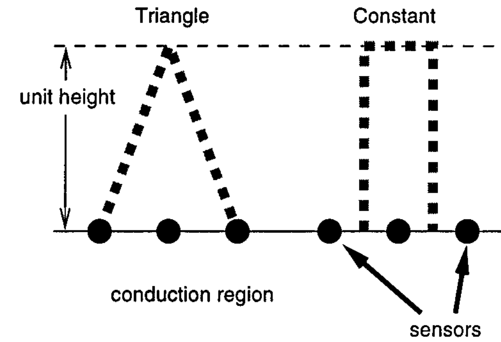

Fig. 3 Triangular patch provides a linearly varying function in space for sensitivity calculations, whereas the constant patch provides simple implementation but too much bias.

where $\boldsymbol{X}$ is the sensitivity matrix; its components are given by $X_{ij} \equiv \partial T_{0i}/\partial q_{0j}$. Note that because the conduction problem is nonlinear due to temperature-dependent properties and multiple conduction layers, a correction Δq to the current estimate q_0 was sought. The estimate is expressed as $\boldsymbol{q} = \boldsymbol{q}_0 + \Delta \boldsymbol{q}$, where the estimate at the current iteration is $\boldsymbol{q}_0$. After manipulation, the flux correction can be expressed as a linear set of equations given by

$$\boldsymbol{X}^T\boldsymbol{X}\Delta\boldsymbol{q} = \boldsymbol{X}^T(\boldsymbol{Y} - \boldsymbol{T}_0) \tag{4}$$

where $\boldsymbol{T}_0$ is the calculated temperature for the current iteration.

The sensitivity matrix $\boldsymbol{X}$ in the formulation for the solution [Eq. (4)] is found by perturbing the estimate of the current iteration and recording the temperature change. However, the perturbation that is performed must be chosen with care. As described in a previous paper,[17] in some respect, the sensitivity patch determines the amount of bias that is introduced into the solution. In other words, where and how the current estimate is perturbed can affect the stability and accuracy of the solution significantly.

Two different type of patches were used for this analysis. The linear patch and the constant patch correlate to a linear function specification and a constant specification method in space, respectively. An example of the linear patch is shown in Fig. 3, where the extent of the patch is at the minimum of one sensor on each side. Note that, to increase the bias, the span of the patch can be stretched to include two sensors on each side. In this case, a linear distribution from one (at the center) to zero (two sensors away from the center) would constitute the perturbation. Note that the estimate obtained for the sensor in question (the sensor in the center) must be distributed over the patch region in accordance with the weighting dictated by the patch. The second patch, also illustrated in Fig. 3, is a piecewise constant function. The added bias can be increased in this case as well by expanding the extent of the patch to cover more sensors.

Regularization

The regularization method is a minimization problem where the difference between the temperature measurements and a calculated temperature (residual) is minimized as for the function-specification method. However, the formulation is derived from a biased least-squares fit of the measured, $\boldsymbol{Y}$, and calculated, $\boldsymbol{T}$, temperature history data (called the objective function S). The objective function is given as

$$S = (\boldsymbol{T}_0 - \boldsymbol{Y})^T(\boldsymbol{T}_0 - \boldsymbol{Y}) + \alpha\boldsymbol{H}^T\boldsymbol{H}\boldsymbol{q}_0 \tag{5}$$

where the bias $\alpha\boldsymbol{H}^T\boldsymbol{H}\boldsymbol{q}_0$ is a function of the current flux estimate being optimized (to be discussed later) and $\boldsymbol{T}_0$ contains the calculated temperatures as a result of the flux estimate at the current iteration. For this formulation, the temperature must be written in its Fourier expansion taking only the first-order term, and the flux is represented in terms of a correction as before. The linear set of equations that represents the flux correction is

$$[\boldsymbol{X}^T\boldsymbol{X} + \alpha\boldsymbol{H}^T\boldsymbol{H}]\Delta\boldsymbol{q} = \boldsymbol{X}^T(\boldsymbol{Y} - \boldsymbol{T}_0) - \alpha\boldsymbol{H}^T\boldsymbol{H}\boldsymbol{q}_0 \tag{6}$$

This methodology is also used in a sequential manner.

The regularization that is added as bias to stabilize the solution is usually first order, which means that the type of bias forces the estimates toward a line. This means that the coefficients of the regularization term $\boldsymbol{H}$ look like a difference formulation of a first derivative. The regularization parameter α is chosen so that there is enough bias to stabilize the problem without destroying the solution with too much deterministic error. The selection of the parameter is accomplished, with trial and error, by requiring S to be on the same order as the noise in the data. Of course, determining the nature and amount of noise in the recorded signal requires additional estimation and will be discussed later.

Modifications of the regularization method for problems with surface temperature measurements were suggested by Walker and Scott[17] and are implemented in this work. This work represents the first attempt to apply the new method to a two-dimensional problem. It was suggested that because the measurements are on the surface, future time steps are unnecessary. To provide bias, however, more than a single time step must be examined. The approach, then, is to examine the preceding flux and the current flux in the regularization term and to ignore the temperature difference at the preceding time step in the residual term. The problem has been simplified because the correction is only at a single time step.

To implement this method for a multidimensional problem, we must look at several adjacent sensors simultaneously. For the patch in this case, though, we will use the configuration that introduces the least bias so that this can be controlled by the regularization. The patch chosen was the linear patch that spans a single sensor on either side. Now the multidimensional approach has two separate levels of regularization that could be added, the first being regularization in space and the second being regularization in time. Therefore, the flux correction can be found using the linear equations as

$$\left[\boldsymbol{X}^T\boldsymbol{X} + \alpha_s\boldsymbol{H}_s^T\boldsymbol{H}_s + \alpha_t\right]\Delta\boldsymbol{q} = \boldsymbol{X}^T(\boldsymbol{Y} - \boldsymbol{T}_0) - \alpha_s\boldsymbol{H}_s^T\boldsymbol{H}_s\boldsymbol{q}_0 - \alpha_t(\boldsymbol{q}_{0i} - \boldsymbol{q}_{i-1}) \tag{7}$$

where the subscripts s and t are the spatial and temporal regularization, respectively. Note that the the temporal regularization is written in terms of the previous known heat flux. The regularization in the spatial direction is a finite difference representation of the first derivative of flux[20] for a complete description on the designation of first-order regularization.

Results

Test cases were developed to verify the solutions obtained from the inverse procedures. A spatial step heat flux was examined to simulate the localized heating found in the shock–shock interaction data. Because the exact data are known, errors can be calculated instead of residuals as with experimental data. The spatial step ramped from 0 to 100 W/cm^2 in 0.4 s, which is comparable (before normalization) to the experimental data. Two limiting configurations are presented where the step lies on a sensor (case 1) and where the step lies directly between two sensors (case 2). The rms of the errors reported in Table 1 were calculated and compared for the one-dimensional and two-dimensional techniques.

Four experimental runs are examined. Each case represents the same experimental setup in that the sweep angle of the instrumented cylinder in the flow stream is $\lambda = -25$ deg as documented by Berry and Nowak.[8] Two runs (14 and 36) use a Macor model, whereas runs 58 and 60 use the Upilex model with a higher sensor resolution. (The runs will be referred to as Macor-1, Macor-2, Upilex-1, and Upilex-2, respectively.) A typical transient response of the sensor that receives the largest heating is indicated by the temperature shown in Fig. 4. A sudden jump in heating rate corresponds to the steep rise in temperature as the shock–shock interaction develops.

Table 1 Error in the flux estimate of the two test cases for the one-dimensional and two-dimensional methods

	Error (rms), W/cm^2	
Method	Case 1	Case 2
Two-dimensional	0.485	0.204
One-dimensional	3.253	3.305

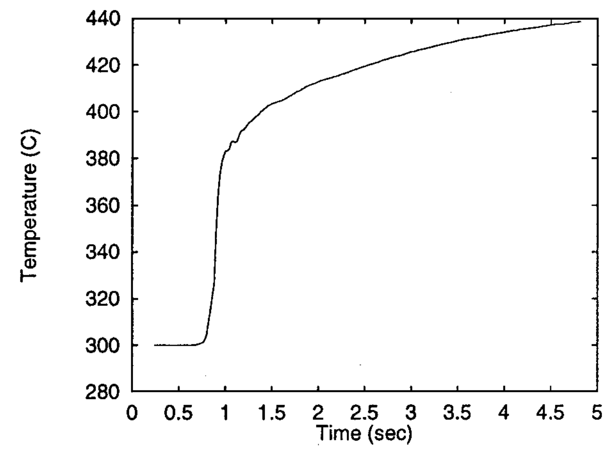

Fig. 4 Temperature response at the sensor of peak heating for Macor-1.

Pivotal to the description of the inverse solution methodology is the characterization of the noise in the measurements and the selection of the regularization parameter. Realize that bias in the measurements cannot be tolerated by the inverse technique that is employed here. Therefore, errors such as lag due to thermal constants of sensors are not addressed. Note, however, that the thickness of the sensors is on the order of microns and is not expected to influence the measurement significantly. Random noise, on the other hand, can be characterized and incorporated into the method. Without knowing much about the measurements, it was assumed that a normally distributed random error with a variance of 5 K exists in the signal. By trial and error, the regularization parameter that results in the optimal solution was $\alpha = 0.0001$ K^2 cm^4/W^2.

The heat flux due to the shock interaction is a function of the surface temperature of the model [$q = C_h/(T_{aw} - T_w)$], where T_w is the temperature of the surface of the model and is dependent on material properties. The normalized heat transfer coefficient C_h is relatively independent of surface temperature. Note that the normalized heat flux incident on both models (Macor and Upilex) should be similar because the flow and resulting heat rates are independent of the model and material properties. It is for this reason that the normalized heat flux ($C_h/C_{h\,\mathrm{ref}}$) will be the reduced quantity and referred to as the heat flux.

Despite the independence of the heat flux on temperature, there is some uncertainty in the properties and geometries of the materials being used. For the Upilex case, the glue layer, which is assumed to be 0.001 in. thick compared to the 0.002-in.-thick Upilex, was ignored as a conducting layer for this analysis so that the results could be compared to the one-dimensional estimates performed previously that also ignore this layer. As a result, the estimate, which depends on the material properties, could contain some bias as a result of the inaccurate model. Therefore, we should not expect exact agreement between the Macor and Upilex models.

Even though the model is curved, the analysis is performed assuming the test article can be modeled as a flat plate. This type of analysis does not hold; an accurate analysis would have to account for the effects of curvature on the conduction. In fact, Buttsworth and Jones[21] suggested a correction factor for curved models. The intent of this research, however, is to examine the validity of an inverse technique over standard forward techniques. Because a flat plate assumption was used for previous forward analysis, the same assumption was used for our inverse analysis so that curvature effects could be eliminated as a cause for variation between the two types of solution methods. Note that an inverse method can incorporate a forward model with any level of complexity, including curvature effects.

At 2 s (when the shock is considered steady), the spatial distributions of the instantaneous heat flux estimates are compared. Macor-1 and Macor-2 in Figs. 5 and 6, respectively, show the increased

Table 2 Average load at 2 s

Run	Two-dimensional	One-dimensional
Upilex-1	2.09	2.19
Upilex-2	2.23	2.18
Macor-1	1.27	1.24
Macor-2	1.62	1.56

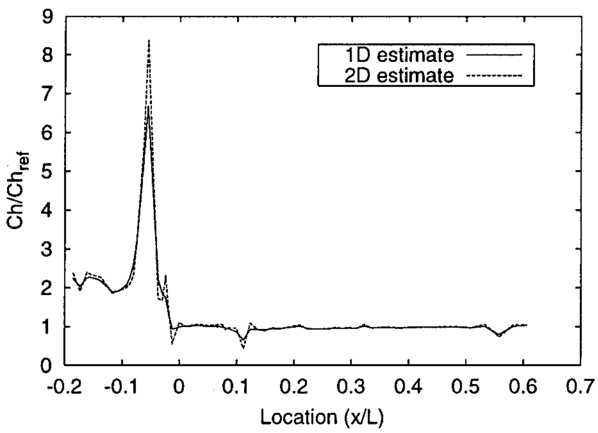

Fig. 5 Comparison of two-dimensional and one-dimensional estimate of Macor-1 at 2 s.

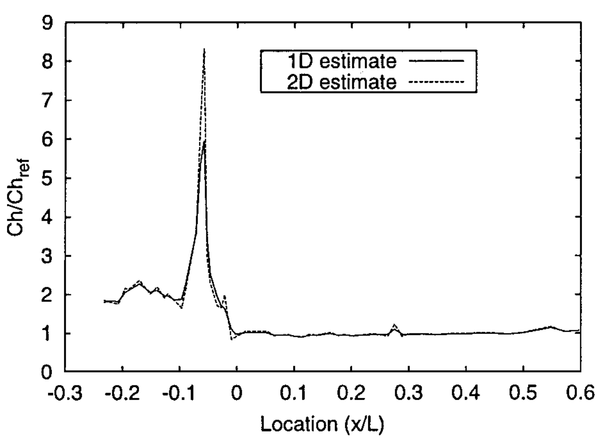

Fig. 6 Comparison of two-dimensional and one-dimensional estimate of Macor-2 at 2 s.

estimate of the two-dimensional model over the one-dimensional case. The heat flux values have been normalized to remove effects of initial temperature and model temperature. (For a complete discussion of the effects of the tunnel and data reduction method, refer to Berry and Nowak.[8]) Note that the value of the heat flux for the two-dimensional analysis is nearly identical between the two runs as expected; the one-dimensional analysis does not predict similar heat fluxes between the two runs. This effect is an artifact of the location of the fluid structures that induce the heat fluxes relative to the sensors. Because the heat fluxes are highly localized, it was shown by Walker and Scott[17] that the location of the peak heat flux relative to the sensor is significant in being able to resolve the magnitude of the heat flux accurately. Conduction effects in this case become significant and must be modeled as in the two-dimensional case.

To verify that the two-dimensional analysis does not introduce additional bias into the solution, we can compare the load integrated over the surface. If the temperature is representative of the energy in the system, both the one-dimensional technique and the two-dimensional technique should be similar. Table 2 shows that the difference in average load is from 3 to 5% and that neither is consistently larger than the other. This indicates that the two-dimensional method is capable of adjusting the distribution to account for lateral conduction without adding bias to the solution.

One question that remains after examining the data concerns sensor spacing. The design of an experiment to recover heat fluxes from temperature measurements will rely heavily on the features of the data reduction method. It was shown that missing sensors as in Upilex-1 can adversely affect the results. However, the sensor spacing of the Macor models is larger but does not seem to be affected in the same way. The required sensor spacing, therefore, is material dependent. For example, because the diffusivity of Macor is 3.5 times that of Upilex, we can expect the penetration depth of the Upilex to be much smaller than that of the Macor. Therefore, the temperature gradients between the sensors could be more pronounced in the Upilex case resulting in a different spacing requirement.

In addition to being material dependent, adequate sensor spacing is also determined by jet size. CFD calculations[3] place the size of the fluid jet on the order of the spacing of the sensors. Conservatively, the size of the peak heating is assumed to be similar to the size of the impinging jet. Therefore, it is difficult to determine the shape and actual magnitude of the peak heating region because we may only have a maximum of two or three sensors that fall in this region. Because the location of the jet relative to the sensors cannot be carefully controlled, the temperature distribution between sensors cannot be accurately characterized. However, one-dimensional methods that ignore lateral conduction should provide even less accurate results because the gradients are ignored completely.

We are able to glean additional knowledge about the two-dimensional method vs the one-dimensional method. The flow structure from schlieren images[8] near the impingement indicates a series of expansion and compression waves moving away from the impingement site. As a result, we would expect to see spatial oscillations in heat flux away from the peak location. Because the two-dimensional method can include conduction effects, the oscillating heating rates can be recovered adjacent to the peak.

The results of the two runs with slightly higher sensor resolution (Upilex model) provide similar information to that of the Macor model. Because of the measured sensor spacing and the different thermal properties on the surface, the Upilex runs required use of the regularization method that allows the bias to be controlled more closely. The two-dimensional estimates from the Upilex model are again higher in the peaks and lower in the valleys, as shown in Figs. 7 and 8.

Some gauge signals were lost during the Upilex-1 experiment in the region of the maximum heating. As a result, the two-dimensional method has less additional knowledge of the lateral conduction to influence the estimate. The results reduce to a one-dimensional method as the temperature information is reduced. This phenomenon indicates that the two-dimensional inverse approach incorporates significant lateral conduction effects and verifies the two-dimensional method in a limiting case. However, we cannot determine whether the given sensor spacing is adequate to fully resolve the effect accurately.

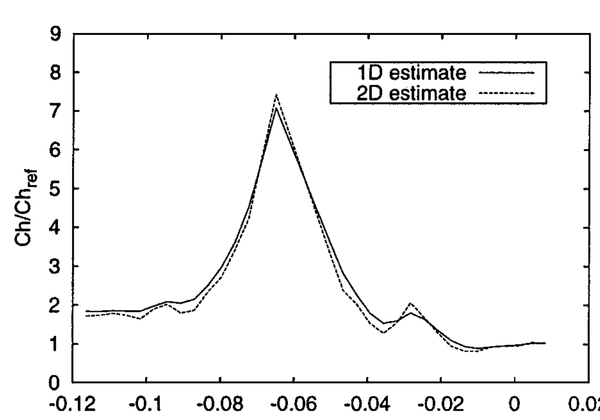

Fig. 7 Comparison of two-dimensional and one-dimensional estimate of Upilex-1 at 2 s.

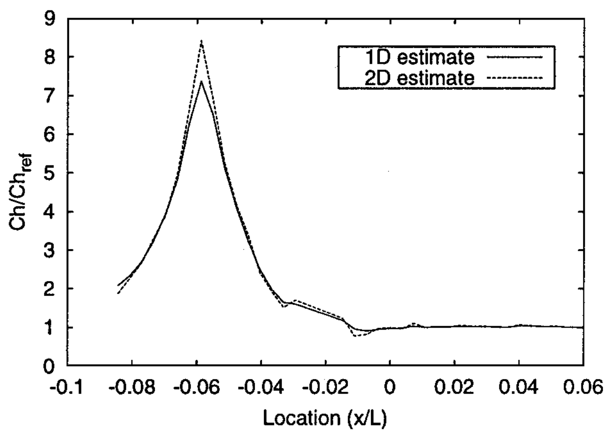

Fig. 8 Comparison of two-dimensional and one-dimensional estimate of Upilex-2 at 2 s.

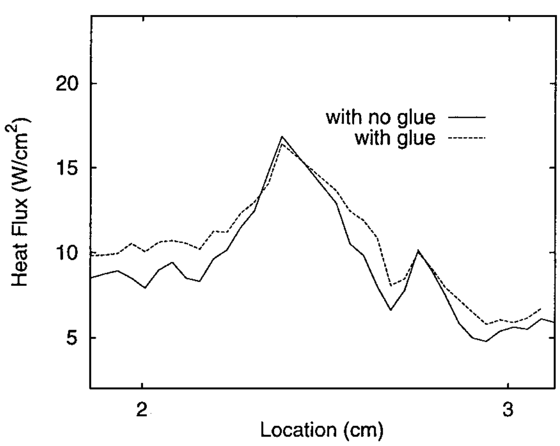

Fig. 9 For Upilex-1 the glue layer was initially ignored and then included in the model.

A comparison of Upilex-2 with the two Macor runs results in similar peak heating rates. This agreement furthers our confidence in the solution technique because the heat flux coefficient should be material independent.

In an attempt to address the question of whether the modeling of the glue layer affects the estimate, Upilex-1 was analyzed using a numerical model with glue and one without glue. Because the properties are largely unknown, material properties for typical high-temperature epoxy were used. The goal in this analysis was to determine if the modeling of the glue layer would have a significant effect. The results are material dependent but suggest that a more accurate and complex model is required for accurate estimation of heat fluxes. In both cases, the two-dimensional inverse approach was employed. As expected, in Fig. 9 the estimate is higher when the glue is considered. Note that the chosen properties of the glue tend to smooth the peak.

In an attempt to validate the estimates, the conduction solutions using the one- and two-dimensional estimates were calculated. These resulting temperature responses were then compared to the original measurement to obtain residuals. Shown in Fig. 10 are the residuals for Upilex-2 at a time of 2 s. [Note that the maximum $\Delta T = (T_{\text{measured}} - T_{\text{calculated}}) \approx 60$ K.] A positive residual means that the estimate overshot the target temperature. Ideally, we would like the residuals to lie close to zero and within the measurement noise. It is immediately clear that the one-dimensional method introduces a great deal of error into the estimate, whereas the two-dimensional approach tends to match the measurements better. This finding is not surprising despite the addition of bias into the two-dimensional inverse technique because the one-dimensional approach is biased by ignoring lateral conduction effects.

Table 3 Residual of rms of the estimates at 2 s, in Kelvin

Run	Two-dimensional	One-dimensional
Upilex-1	4.74	93.2
Upilex-2	9.98	94.9
Macor-1	2.13	17.6
Macor-2	2.18	17.4

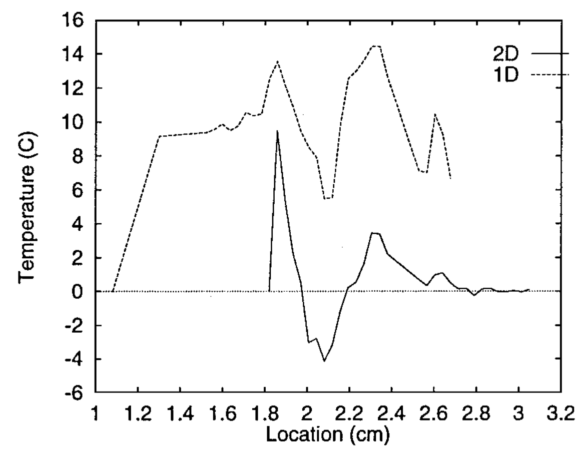

Fig. 10 Residual is the difference between the measured and calculated temperatures for Upilex-2 at 2 s.

To obtain a quantification of how well the estimation methods performed, we calculate the rms residual. This is simply a measure of the average of the difference between the measured and calculated temperatures. Table 3 shows a striking difference between the one- and two-dimensional estimates for all models. The differences between the residuals of the Upilex and Macor models are a result of the discrepancy mentioned earlier, but exist partly because a different conduction model had to be employed. The varying material properties cause the sensitivity of the problem to change. Ultimately, this results in less confident solutions as demonstrated by the larger residuals. Following Haftka et al.,[22] this result suggests that the entire experimental setup can and should be designed to maximize the value of any measurements made and to minimize the effects of uncertainties in the experiment.

Conclusions

The results clearly identify the two-dimensional technique as producing more accurate results. Therefore, lateral conduction within the model must be considered when temperature gradients exist between adjacent sensors. When the heating rate is large and localized, the energy that diffuses laterally will be ignored by one-dimensional estimation techniques. The two-dimensional inverse technique demonstrates its ability to account for direct heating as well as diffusive heating by estimating a significantly higher peak heating rate. Despite the improved accuracy of the inverse approach with its ability to track lateral conduction effects, the estimates cannot account for inadequate experimental data. Because the location and size of the peak heating region is considered to be on the order of the sensor spacing, the apparent lack of data reduces the accuracy of the estimates.

To qualify the increased accuracy claims, note that inverse techniques allow for better modeling of phenomena. They cannot, however, compensate for an inadequate quantity of measurements. In other words, inverse methods make better use of the existing data, but do not increase the amount of information in measurements. What inverse techniques do provide is the ability to incorporate additional measurement information and the ability to incorporate physically accurate (complex) mathematical models.

Acknowledgment

The authors would like to thank NASA Langley Research Center for awarding Graduate Student Research Program Fellowship NGT–51249, which made this work possible.

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